Practical Radiometry

It is often necessary to estimate the response of a camera under given lighting conditions, or perhaps to estimate lighting requirements for a particular camera. To assist with these calculations a number of examples are given below for common problems. It should be recognized that very accurate calculations are difficult; it is often better to start with a simple estimate and then fine tune the set up experimentally. A conversion table is provided to enable different measurement units to be compared; all calculations are carried out using radiometric SI units (see the CCD technology primer in this data book).

1. Magnification and Resolution

The camera system is to be set up such that each pixel in the CCD sensor corresponds to a set feature size on the object. What is the required magnification of the camera? The answer depends on the pixel size, the focal length of the camera lens, f, the distance from the object to the lens, OD and the distance from the lens to the image, ID. (Please see Figure 1.)

The magnification can be given by:

$$m = \frac{\textit{size of image}}{\textit{size of object}} = \frac{ID}{OD} = \frac{f}{OD - f}$$

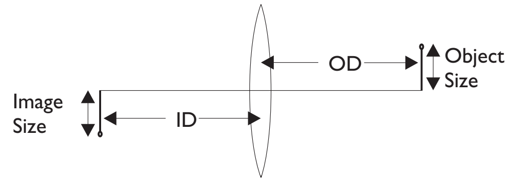
when the focal length of the lens is much smaller than OD this can be simplified to m = f / OD and hence the required OD is given by OD = f / m.

The required magnification is m = 10µm / 1mm = 0.01,

Example: pixel size = 10µm x 10µm
focal length of lens = 55mm
object feature size = 1mm

hence OD = 55 / 0.01 = 5.5m

Figure 1. Example 1



2. Camera Output with Monochromatic Light Source

I have a monochromatic light source, such as a green phosphorous screen in an image intensifier. (Please see Figure 2.) What is the output from the camera? The answer depends on the responsivity of the camera, the camera lens and the distance, from the object to the lens. It conveniently turns out that for an image formed by an optical system in which the losses are negligible, the radiance of the image is the same as that of the object. This may sound counterintuitive since it is well known that when a lens is used to focus an image on a screen its brightness to the observer increases as the magnification is decreased. However, if the image is observed directly by the eye, its brightness appears unchanged. This is because when the magnification is decreased, the flux per unit area is increased, but the total solid angle is also increased in such a way that the radiance remains constant. Therefore, given the radiance of an object $L_{(object)}$, the radiance of the image is essentially the same, subject to losses in the lens characterized by T_r, the radiance transmittance. In order to calculate the irradiance at the sensor it is then a simple matter to multiply the radiance by the solid angle subtended by the lens at the sensor. This will depend on the diameter of the lens, and if an iris is employed to stop down the lens, it will depend on the diameter of the iris, d. The solid angle subtended at the sensor is the ratio of the active lens area ($1/4\,\pi\, d^2$) to ID. Hence, by expressing ID in terms of the magnification, and the iris diameter in terms of the f-number, the irradiance, of the sensor $E_{(sensor)}$ is given by;

$$E_{(sensor)} = \frac{T_r L_{(object)} \pi}{\left[2(f/\#)(m+1)\right]^2} \left(W \cdot m^{-2}\right)$$

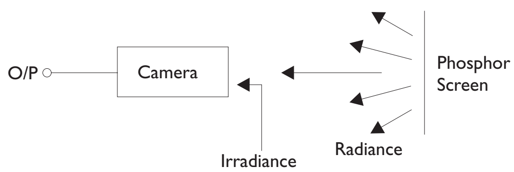
when the magnification is small, this simplifies to

$$E_{(sensor)} = T_r\, L_{(object)}\, \pi / (2(f/\#))^2$$

Example: radiance(object) = 2 µW cm^{-2} Sr^{-1}
f/# = 1.4
assume a perfect lens T_r = 1
required magnification = 0.01

The irradiance at the sensor is therefore = 0.8µW/cm^2.

Figure 2. Example 2



Example:

line rate of camera (LVAL) = 1.5kHz
wavelength = 546nm

The output from the camera will depend on its responsivity at the wavelength of interest and the integration time of the sensor.

First, we refer to the performance characteristics of the camera to find the peak responsivity. Consider an example camera giving 126 DN/(nJ/cm^2), measured at 800nm. This determines the output of the camera for a given radiant energy density. The next step is to relate this information to the wavelength of interest. We refer to the spectral responsivity curve for the camera's sensor. The responsivity at the wavelength of interest (546nm) is found to be lower than that at the wavelength used to characterize the camera (800nm) by 58%. Hence the responsivity for the camera at 546nm is reduced by 58% to 53DN/(nJ/cm^2) for the analogue and digital cameras respectively. The camera is to be operated at a line rate of 1.5kHz, corresponding to an integration time of 0.67ms. The radiant energy density at the sensor is simply the irradiance (a power density) multiplied by the integration time, which in this case gives 0.54nJ/cm^2. Therefore the output from the camera (without a lens) would be 29DN.

3. Convert photometric to radiometric units - Monochromatic

Suppose that in the last example the source had been characterized by a luminosity of 4 ASB (apostlib), a photometric unit. Before calculating the camera output it is necessary to convert this to an SI radiometric unit. We refer to conversion Table 1 to find that 1 ASB is equivalent to 0.318 lux / Steradian, which can be converted to radiance with the knowledge that 1 lux is equivalent to 1.49µW/m^2 of monochromatic light at 546nm. Hence, 4 ASB becomes 1.9µW cm^{-2} sr^{-1}, which is roughly the same as that used in the previous example. The same basic idea is valid for the conversion of other photometric units.

4. Illumination of Object - Monochromatic

Consider the case where the phosphorous screen in example 2 were replaced by a diffuse plane, illuminated by a monochromatic light source. The plane surface, or object is face-on to the camera, while the normal to the surface makes an angle φ with the light source. If the source is a point source it will radiate in all directions. In reality, the light source will probably incorporate a back reflector and a diffuser to provide an extended, uniform illumination source. It is important to ensure that the light source is 'aimed' towards the object otherwise the irradiance can become very sensitive to the angle. Even in this case, the irradiance is reduced as φ grows larger. For an object a distance LS from the light source of diameter s, the average irradiance over a small area of the object is given by

$$E_{(object)} = L_{(source)} COS(\varphi) \frac{\pi}{4} \left(\frac{s}{LS} \right)^2 \; (\mathrm{W.m^{-2}})$$

Assuming that the object reflects this equally into a full hemisphere (Lambertian surface) it can be shown that the radiance of the object with a reflectivity R, is given by

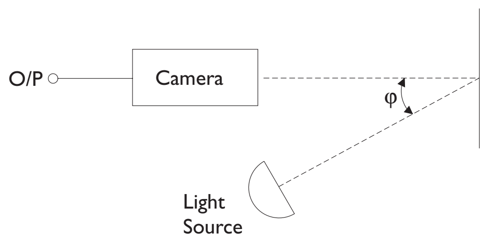
$$L_{(object)} = E_{(object)} \frac{R}{\pi} \; (\mathrm{W.m^{-2}.sr^{-1}})$$

This value is used in place of the radiance of the light source in Example 2.

Example: L(source) = 2µW cm^{-2} sr^{-1}
R = 56%
s = 5cm
LS = 0.5m
φ = 45 degrees

In this example (please see Figure 3), the radiance of the object = 2 μW cm^{-2} sr^{-1}, giving the same result as in Example 2.

Figure 3. Example 4



5. Light Source Requirements - Polychromatic

When the light source is polychromatic the full spectral response of the camera has to be taken into account. In simple terms, this amounts to repeating the monochromatic calculation for each wavelength in the range of interest. To do this accurately involves integration over the spectrum, which is probably more conveniently carried out by computer. Nevertheless, it is often sufficient to arrive at an estimate based on a simplified approach. This is accomplished by dividing the spectrum into a small number of discrete intervals, λ_l-λ_2, λ_2-λ_3,... λ_n-$\lambda_{(n+1)}$. Within each interval it is assumed that the value of radiance etc. is constant and equal to the exact value it would have at the midpoint of each wavelength interval. When approximating any integrals over the spectrum it is a simple matter to take the value at the midpoint of each interval, and add them all up. Suppose then, that the user wishes to determine the lighting requirements for a given camera. The output will depend on the spectral output of the light source, the spectral reflectivity of the object and the spectral response of the camera. Furthermore, the conversion of photometric to radiometric units is complicated by the need to use the CIE Luminous Efficiendy curve (of the human eye). As this is very application specific, please contact DALSA for further help in deciding on your requirements.

Example: what is the camera output when the light source is a tungsten halogen lamp, illuminating a flat piece of green (e.g. snooker table) velvet as it comes off the end of the manufacturing process?

It is probably fair to assume that the velvet is uniform, of fine weave so that it may be treated as a plane homogenous diffuse scattering surface. There are now three extra factors to consider, the spectral output of the lamp, the spectral reflectivity of the velvet and the spectral response of the sensor. Color aberrations and non-uniform transmission losses by the lens are assumed to be negligible. We also assume that the velvet has been dyed with an advanced formula that is known to give a fairly uniform reflectivity of 20% across the green portion of the spectrum (500nm - 600nm), dropping to 2% otherwise. The light source is 5cm in diameter, operated at a color temperature of 3200K which is known to produce a spectral irradiance at the velvet of 0.1μW/cm^2 nm^{-1} at 650nm when positioned 0.5m away. The example camera used operates at 10 frames per second with an f/1.4 lens (perfect transmission) and a magnification of m = 0.01.

The first step is to discretize the spectrum into a number of convenient intervals, which for this example will be restricted to 400-500nm, 500-600nm, and 600-700nm. The midpoints are thus 450nm, 550nm and 650nm. The next step is to calculate the irradiance at the light source at these wavelengths.

Assuming it to be a perfect Black Body radiator, the spectral radiance (per wavelength interval, nm) of a lamp at temperature T (K), for a wavelength λ (nm), can be shown to be given by Planck's Law [W.J. Smith, "Modern Optical Engineering, 2nd Edition, p.217],

$$L(\lambda) = \frac{3.741\text{x}10^{19}}{\pi\lambda^5\left[\exp\left(\frac{14388000}{\lambda T}\right)-1\right]} \quad (\text{W cm}^{-2}\ \mu\text{m}^{-1}\ \text{sr}^{-1})$$

This shows why the radiance is a very strong function of the wavelength. From this equation it is possible to deduce that at T = 3200K, for a given radiance at 650nm, the radiance at 550nm is down to 66% and at 450nm it is down to 29%. It is interesting to note that the wavelength at which the radiance is maximum, which is given by Wien's displacement law, λ = 2897800 / T, is λ = 906nm (IR). The irradiance at the velvet is found by integrating the spectral irradiance curve over the wavelength range of interest. In this case we assume that the spectral irradiance is constant in each interval, fixed to the value it has at the midpoint. Thus, in the range 600nm-700nm the irradiance is 100nm times the spectral irradiance at 650nm, giving an irradiance of 10μW/cm^2. Therefore the irradiance at 550nm is 6.6μW/cm^2 and at 450nm it is 2.9µW/cm^2. The reflectivity of the velvet is wavelength dependent, so we use R = 0.2 at 550nm and R = 0.02 at 450nm and 650nm. Using the second equation from example 4 the radiance of the velvet can be calculated as 64nW cm^{-2} sr^{-1} at 650nm, 420nW cm^{-2} sr^{-1} at 550nm, and 18nW cm^{-2} sr^{-1} at 450nm.

Next we calculate the irradiance at the sensor using the same technique shown in example 2. Thus we have to multiply the velvet radiance values for each wavelength by 0.4 to give a sensor irradiance at 450nm of roughly 7.4 nW/cm^2, at 550nm 170nW/cm^2, and at 650nm 26nW/cm^2. The integration time when operating at 10fps is 100ms, hence the energy density at the sensor is is found by multiplying the irradiance by the integration time, yielding 0.7nJ/cm^2 at 450nm, 17nJ/cm^2 at 550nm and 2.6nJ/cm^2 at 650nm. The responsivity of the sensor is also wavelength dependent. Assume the peak responsivityof the 8-bit example camera is 10 DN/ (nJ/cm^2), measured at 800nm. At 650nm its responsivity is reduced to 7.8DN/(nJ/cm^2), at 550nm to 5.6DN/(nJ/cm^2) and at 450nm to 2.0DN/(nJ/cm^2). This gives an output from the camera of 1.4DN at 450nm, 96DN at 550nm and 20DN at 650nm**.** Adding these together, the predicted output for the camerais roughly 117DN. It is instructive to note that the contribution from the 'green' part of the spectrum dominates. This is mostly because the velvet is a strong reflector at this wavelength, but it is exacerbated by the strong sensor responsivity at the same wavelength. This example is over simplified, in practice a more detailed calculation is required. It is best performed numerically and should serve only as a guide before measuring the output experimentally. This example shows that for the given lighting conditions the output of the camera is only approximately one-half of its saturation voltage.

Table 1. Useful conversion factors

1 ASB (apostlib)	=	0.318 lux Sr^{-1}
1 Footcandle	=	1 Lumen / Ft^2
1 Footcandle	=	10.764 meter Candles
1 Candle	=	1 Lumen/Steradian
1 Candle / m^2	=	3.142 x 10^{-4} Lambert
1 Lambert	=	2.054 Candle / in^2
1 Lux	=	1 Meter Candle
1 Lux	=	1.49µW/cm^2 at 540nm
1 meter Candle	=	1 Lumen / m^2
1 Lumen	=	average light emission of one candle into a unit solid angle (in steradians)

Table 2. Radiometric Units

Symbol	Name	SI units
L_e	Radiance	W m^{-2} sr^{-1}
E_e	Irradiance	W m^{-2}
I_e	Intensity	W sr^{-1}
F_e	Flux	W
Q_e	Energy	J

Table 3. Photometric Units

Symbol	Name	Units
L_v	Luminance	cd m^{-2}
E_v	Illuminance	lux (lx)
I_v	Intensity	candela (cd)
F_v	Flux	lumen (lm)
Q_v	Energy	Talbot

Table 4. Conversion Between Photometric Luminance Units

Multiplication Factors FROM	Cd/cm² (stilb)	cd/ft²	Cd/m² (nit)	Lmb	Ft.Lmb	Mtr.Lmb (apostlib)	Threshold of Unaided Vision	Typical Level (clearsky)
cd/cm² (stilb)	1	A	10000	π	πA	10000 π	10^{-7}	0.3
cd/ft²	1/A	1	B	π/A	π	πB	10^{-4}	300
cd/m² (nit)	10^{-4}	1/B	1	$10^{-4}\pi$	π/B	π	10^{-3}	3000
Lmb	1/π	A/π	$10^{-4}/\pi$	1	A	10^{-4}	3×10^{-7}	1
Ft.Lmb	1/πA	1/π	B/π	1/A	1	B	3×10^{-4}	1000
mtr.Lmb	$10^{-4}/\pi$	1/πB	1/π	10^{-4}	1/B	1	3×10^{-3}	10000